# MATH TOOLBOX

## The Case of the **RADIOACTIVE FOOTPRINTS By Mitch Ricketts**

Math Toolbox is designed to help readers apply STEM principles to everyday safety issues. Many readers may feel apprehensive about math and science. This series employs various communication strategies to make the learning process easier and more accessible.

#### **Radioactive materials are common**

in many workplaces. For example, medical facilities maintain radioisotopes for diagnosis and treatment; manufacturers acquire radioactive sources to sterilize products and identify hidden flaws through nondestructive testing; construction and agricultural establishments keep radioactive devices to measure moisture content and density of materials; research laboratories use radioactive isotopes to trace the movement of gases, fluids and solids; and power companies

house radioactive materials in reactors to provide electricity for millions of customers worldwide.

In these and other applications, safety professionals must protect against uncontrolled release of isotopes and ionizing radiation. Figure 1 illustrates the extensive measures that may be required when even small amounts of radioactive materials are inadvertently released into the environment.

It is impossible to completely eliminate radiation exposure because we all receive

## FIGURE 1 **RADIOACTIVE CONTAMINATION, COLUMBIA, MO**



cells may repair themselves. If damage is serious or widespread, cells may die or DNA may be changed in ways that prevent cells from reproducing normally. Intense exposures may cause acute radiation syndrome and death. Less intense but chronic exposures may lead to cancer, cataracts and many other illnesses. It is not possible to specify a safe level of exposure to ionizing radiation, so it is generally recommended that exposures be kept as low as reasonably achievable.

#### **Radioactive Decay**

Atoms emit radiation during a process called radioactive decay. To understand the decay process, it is helpful to review some basic concepts of atomic physics.

limited doses from naturally occurring

rays arriving from space. Nevertheless,

decades of research have shown that

life-sustaining molecules and creates

toxic by-products. If damage is minor,

radioisotopes on Earth and from cosmic

ionizing radiation is hazardous to health.

Within the body, radiation breaks apart

Atoms are small particles of matter. Every atom belongs to a particular element such as hydrogen, carbon or gold. More than 90 naturally occurring elements have been identified and many more have been created in laboratories. At the heart of every atom is the nucleus (see Figure 2). Within the nucleus reside particles called protons that have a positive electrical charge. Other particles in the nucleus are called neutrons. As their name implies, neutrons are electrically neutral, meaning they have no net positive or negative electrical charge. The nucleus is surrounded by electrons—each with a negative charge. Electrons are imagined to reside in particular energy levels or shells. Under standard conditions, each lower energy level fills with electrons before the next higher energy level can exist.

The number of protons in a nucleus determines the chemical element to which an atom belongs. For instance, all atoms of the element hydrogen (H) contain one proton, while all atoms of the

nation Incident," by P.C. Ashbrook, 2011, Journal of Chemical Health and Safety, 18(3), 5-9, https://doi .org/10.1016/j.jchas.2010.11.001.

element helium (He) contain two protons. Lithium (Li), which always contains three protons, is depicted in Figure 2. The heaviest elements have about a hundred protons.

While all atoms of a particular element have the same number of protons, the number of neutrons may vary. The isotope number represents the number of protons plus the number of neutrons. For instance, all atoms of carbon (C) have six protons; however, the stable isotope carbon-12 ( $^{12}$ C) has six protons plus six neutrons (6 + 6 = 12), while the unstable isotope carbon-14 ( $^{14}$ C) has six protons plus eight neutrons (6 + 8 = 14).

A radioactive isotope is an atom with an unstable nucleus. This type of atom tends to stabilize itself through radioactive decay, a process that makes changes in the nucleus (Figure 3). For instance, during alpha decay, the isotope's nucleus becomes more stable by ejecting a positively charged alpha particle, consisting of two protons and two neutrons.

In beta decay, the nucleus stabilizes by ejecting a beta particle. During betaminus ( $\beta^{-}$ ) decay, the nucleus ejects a negatively-charged beta particle called a negatron (with the same mass and charge as an electron). Beta-plus ( $\beta^{+}$ ) decay, in contrast, occurs when the nucleus ejects a positively charged beta particle called a positron (with the same mass but opposite charge as an electron). An antineutrino accompanies beta-minus decay, while beta-plus decay produces a neutrino. In OSH, we ignore the antineutrinos and neutrinos since they rarely interact with ordinary matter.

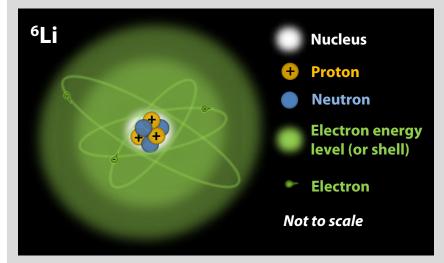
Gamma rays are not particles; instead, they represent a form of electromagnetic radiation. Gamma radiation is usually emitted while a nucleus is in an excited state after alpha or beta decay. Gamma rays are invisible to humans and can pass through many materials.

Neutron radiation occurs when a neutron is emitted from the nucleus of an atom. Neutron radiation is unusual on Earth; however, significant amounts of neutrons may be produced during the decay of a few radioisotopes such as synthesized curium-248 (<sup>248</sup>Cm) and californium-252 (<sup>252</sup>Cf). Otherwise, free neutrons on Earth are associated mainly with nuclear reactors and certain sealed-source testing equipment such as soil-moisture probes.

X-rays are not normally associated with natural radioactive decay. Instead, they are emitted outside the nucleus

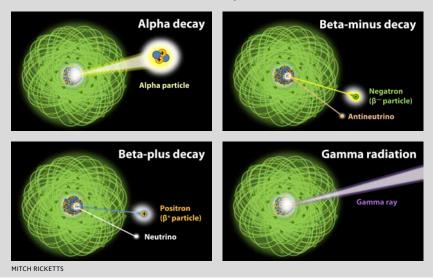
## FIGURE 2 STRUCTURE OF AN ATOM

Lithium (Li) atoms always contain three protons. The illustrated atom represents the isotope lithium-6 (<sup>6</sup>Li) since there are three protons and three neutrons (3 + 3 = 6). The lower (K) energy level contains two electrons, while the next-higher (L) level has just one electron.



## FIGURE 3 RADIOACTIVE DECAY

The most common forms of natural radioactive decay.



when electrons lose energy. The main difference between X-rays and gamma rays is that X-rays come from an atom's electron cloud, while gamma rays originate within the nucleus.

Ions include molecules, atoms and subatomic particles with nonneutral (positive or negative) charges. Alpha, beta, neutron, gamma and X-radiation are called ionizing radiation because they may create ions in any materials they happen to contact, including human tissues.

Again, radioactive decay is a process that makes changes in the nucleus of an unstable atom. Although it is impossible to predict when a particular atom will decay, we can specify the average rate of decay for each isotope. This rate is expressed as the half-life, which is the amount of time it takes, on average, for half the at-

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oms of an isotope in a sample to decay into a different isotope or element.

As an example, the unstable isotope <sup>14</sup>C decays to stable nitrogen-14 (<sup>14</sup>N). <sup>14</sup>C has a half-life of about 5,700 years. Imagine we have a sample consisting of 1 million atoms of <sup>14</sup>C and zero atoms of <sup>14</sup>N. After one half-life of 5,700 years, radioactive decay will leave our sample with about 500,000 14C atoms and 500,000 <sup>14</sup>N atoms. With the passage of each succeeding half-life, there will be half as many atoms of <sup>14</sup>C as before. Thus, after the second 5,700-year halflife (11,400 years from the beginning), we expect our sample to contain about 250,000 atoms of <sup>14</sup>C and 750,000 <sup>14</sup>N atoms. After many more half-lives, the remaining <sup>14</sup>C will become insignificant and the sample will consist almost entirely of <sup>14</sup>N.

Different isotopes decay at different rates, giving each radioisotope its own characteristic half-life. For instance, uranium-238 (<sup>238</sup>U) has a half-life of about 4.5 billion years, while another isotope of uranium, <sup>235</sup>U, has a shorter half-life of about 700 million years. Some isotopes have much shorter half-lives, such as radon-222 (<sup>222</sup>Rn) with a half-life of less than 4 days and polonium-215 (<sup>215</sup>Po) with a half-life of less than 1/500th of a second.

Radioactivity (or simply activity) is quantified by the rate of radioactive decay, typically in units of Becquerels (Bq), or in the former U.S. customary measure of Curies (Ci). One Bq is equal to the radioactivity of one nuclear decay per second, while one Ci equals a decay rate of 37 billion  $(3.7 \cdot 10^{10})$  per second.

#### How Radioactive Decay May Guide Decisions in OSH

Effective management of radioactive sources often depends on the rate at which radiation decreases over time. For example, the response to the incident described in Figure 1 (p. 30) was based on the time required for the <sup>32</sup>P to decay until its radiation would be indistinguishable from normal background radiation. This same approach determines the decay-in-storage period prior to normal disposal of certain short-lived radioisotopes as allowed by the U.S. Nuclear Regulatory Commission. The decay rate also determines the useful life of radiation-emitting devices such as ionizing smoke detectors and nondestructive testing equipment.

In this article, we will examine radioactive decay equations to assist in the management of radioisotopes. We will begin with the radioactive decay constant,  $\lambda$ , which is calculated from the known half-life of an isotope. We will then use value of  $\lambda$  to calculate the amount of radioactivity, *A*, or the quantity of a radioisotope, *N*, remining after a period of decay.

### Radioactive Decay Constant, $\lambda$

The radioactive decay constant is calculated as a preliminary step for many equations related to the management of radioisotopes. We calculate the decay constant from the isotope's known halflife as follows:

$$\lambda = \frac{\ln(2)}{t_{1/2}}$$

which is approximated by

$$\lambda = \frac{0.693}{t_{1/2}}$$

where:

 $\lambda$  = radioactive decay constant of the isotope; that is, a factor that reflects the rate of decay per unit of time (e.g., per second, per minute, per day)

ln(2) = natural logarithm of 2; approximately 0.693

 $t_{\frac{1}{2}}$  = half-life of the radioactive isotope; that is, the average amount of time it takes for half of the radioactive atoms in a sample to decay into a different isotope or element. The units of time entered for half-life determine the units of time for the radioactive decay constant,  $\lambda$ ; for instance, if  $t_{\frac{1}{2}}$  is entered as seconds, the value of  $\lambda$  will be given per second, and if  $t_{\frac{1}{2}}$  is entered as minutes, the value of  $\lambda$ will be given per minute.

**Calculated example.** The incident described in Figure 1 (p. 30) involved the radioisotope  ${}^{32}$ P, which decays directly to the stable (nonradioactive) isotope, sulfur-32,  ${}^{32}$ S. The half-life ( $t_{1/2}$ ) of  ${}^{32}$ P is known to be about 14.3 days (Laboratoire National Henri Becquerel, 2022). Based on this information, we can calculate the radioactive decay constant,  $\lambda$ , as follows:

**Step 1:** Begin with the equation:

$$\lambda = \frac{\ln(2)}{t_{1/2}}$$

**Step 2:** Insert the known value for the half-life of  ${}^{32}P(t_{1/2} = 14.3 \text{ days})$  and solve for  $\lambda$ . The value of  $\lambda$  will be a very small number and we will use it in further calculations, so we will express the value in scientific notation to preserve as much information as possible. To express a small number in scientific notation, we move

the decimal to the right and multiply by 10 with a negative exponent. That exponent will equal the number of places we moved the decimal; for example, the small number 0.00715 is expressed as  $7.15 \cdot 10^{-3}$  because we moved the decimal three places to the right. Most calculators have a key to display answers in scientific notation (often designated as SCI). In an Excel spreadsheet, the display mode can be changed in the number format dropdown menu.

With the value of the half-life inserted, we solve the equation as follows:

$$\lambda = \frac{\ln(2)}{14.3} = 4.8472 \cdot 10^{-2}$$
 per day

which equals 0.048472 per day. Note: Most calculators have an LN button that will provide the correct answer with keystrokes similar to the following in this case:  $LN(2) \div 14.3 =$ . Alternatively, in an Excel spreadsheet, the proper formula for this example is: =LN(2)/14.3.

**Step 3:** Since the half-life was expressed in days, the value of  $\lambda$  is expressed per day, meaning the radioactive decay constant for <sup>32</sup>P is about  $4.8472 \cdot 10^{-2}$  per day. In later calculations, this result will help us determine the amount of radioactivity remaining at selected time intervals after the spill reported in Figure 1 (p. 30).

**Approximation using the second form of the equation.** The radioactive decay constant may be approximated by inserting 0.693 in place of ln(2) for the equation's numerator, as follows:

$$\lambda = \frac{0.693}{t_{1/2}}$$

We obtain a similar, but somewhat less accurate, answer with this approximation. To demonstrate, insert the half-life of <sup>32</sup>P ( $t_{1/2}$  = 14.3 days) and solve for  $\lambda$  using the approximation:

$$\lambda = \frac{0.693}{14.3} = 4.8462 \cdot 10^{-2}$$
 per day

which equals 0.048462 per day. This approximation may be adequate for some uses; however, the original form of the equation with the natural logarithm is used when accuracy is critical.

Alternate example. Radioactive plutonium-239 (<sup>239</sup>Pu) has been used in nuclear fission devices, including the first atomic bomb tested at White Sands, NM, in 1945 (see Figure 4). Unfissioned atoms of <sup>239</sup>Pu were found along with many radioactive fission products in fallout from the bomb test. <sup>239</sup>Pu decays mainly by emitting alpha particles. The half-life of <sup>239</sup>Pu is about 24,100 years. Knowing the half-life, we calculate the radioactive decay constant,  $\lambda$ , as follows: **Step 1:** Begin with the equation:

$$\lambda = \frac{\ln(2)}{t_{1/2}}$$

**Step 2:** Insert the known value for the half-life of <sup>239</sup>Pu ( $t_{\frac{1}{2}}$  = 24,100 years) and solve for  $\lambda$ :

$$\lambda = \frac{\ln(2)}{24,100} = 2.8761 \cdot 10^{-5} \text{ per day}$$

which equals 0.000028761 per year. **Step 3:** Since the half-life was expressed in years, the calculation indicates the radioactive decay constant for  $^{239}$ Pu is about 2.8761  $\cdot$  10<sup>-5</sup> per year.

#### You Do the Math

Apply your knowledge to the following questions. Answers are on p. 41.

1. Radioactive polonium-210 (<sup>210</sup>Po) is used as a static eliminator in textile mills and other industries. <sup>210</sup>Po emits alpha particles as it decays to stable, nonradioactive lead-206 (<sup>206</sup>Pb). The half-life of <sup>210</sup>Po is about 138.38 days. Since the halflife is expressed in days, calculate the radioactive decay constant,  $\lambda$ , per day.

2. Radioactive <sup>14</sup>C emits beta particles as it decays to nonradioactive <sup>14</sup>N. The half-life of <sup>14</sup>C is about 5,700 years. Since the half-life is expressed in years, calculate the radioactive decay constant,  $\lambda$ , per year.

#### Quantity or Radioactivity of an Isotope After a Period of Radioactive Decay

It is helpful to know the quantity (N) of a radioisotope or the radioactivity (A) remaining after a period of decay. When the radioactive decay constant is known, we may use either of two formulas, as follows:

$$N = N_0 \cdot e^{-\lambda}$$

or

$$A = A_0 \cdot e^{-\lambda \cdot t}$$

where:

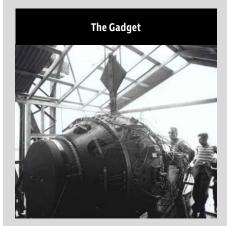
N = quantity of the radioactive isotope after time; usually expressed as the number of radioactive nuclei present or as the mass of the isotope in units [e.g., milligrams (mg), grams (g)]

 $N_0$  = initial quantity of the radioactive isotope in the same units as N

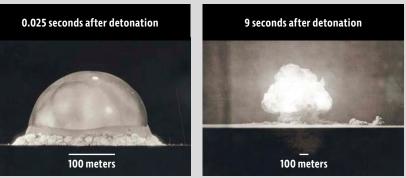
A = radioactivity of the isotope after time; usually expressed as disintegrations per second in units such as Becquerel (1 Bq = 1 disintegration per second) or curies (1 Ci =  $3.7 \cdot 10^{10}$  disintegration per second)

 $A_0$  = initial radioactivity of the isotope in the same units as A

## FIGURE 4 TRINITY NUCLEAR TEST, JULY 16, 1945



The world's first nuclear explosion took place at the Alamogordo Bombing Range in New Mexico. Robert Oppenheimer code-named the test Trinity, and the plutonium device at the core was nicknamed Gadget. Heat from the explosion fused the local soil into a glassy material known as Trinitite. Most of the contaminated soil has been removed from the blast site, but residual radioactivity at ground zero is still about 10 times greater than the region's natural background radiation. Elevated radioactivity is also present in preserved remnants of Trinitite and in the fallout that landed downwind. Fallout-producing atmospheric nuclear tests took place at sites worldwide from 1945 through 1962.



Note. Photos courtesy of U.S. Department of Energy.

*e* = base of the natural logarithm (approximately 2.819291929)

 $\lambda$  = radioactive decay constant of the isotope; that is, a factor that reflects the rate of decay per unit of time (e.g., per second, per minute, per day)

t = elapsed time; expressed in the same units of seconds, minutes or days, as  $\lambda$ 

**Calculated example.** For the incident described in Figure 1 (p. 30), investigators reported the total <sup>32</sup>P radioactivity that spread through campus was 370,000 Becquerel or less (10 microcuries or less in customary measure). This initial radioactivity reflects all of the <sup>32</sup>P that spread through campus during the incident. We cannot know the radioactivity of any single footprint because the initial activities of individual footprints were not measured.

Since <sup>32</sup>P decays directly into nonradioactive <sup>32</sup>S, contamination from <sup>32</sup>P is often managed by isolation until its radiation is no longer distinguishable from natural background levels. Small amounts of <sup>32</sup>P are typically stored for at least 10 half-lives prior to disposal, so we will estimate the total <sup>32</sup>P radioactivity expected to remain after 10 half-lives. The half-life of <sup>32</sup>P is about 14.3 days, so we will estimate the total radioactivity remaining after 143 days ( $10 \cdot 14.3 = 143$ ).

Measures of radioactivity can involve very large numbers; thus, it may be convenient to express these values in scientific notation. To express a large number in scientific notation, we move the decimal to the left and multiply by 10 with a positive exponent. Once again, the exponent will equal the number of places we moved the decimal. This means 370,000 equals  $3.7 \cdot 10^5$  because we moved the decimal five places to the left.

Since we are estimating radioactivity in Becquerel, we will solve for remaining radioactivity, *A*, based on the following data:

•Investigators reported the initial radioactivity of the spilled  ${}^{32}$ P was  $3.7 \cdot 10^5$  Bq or less; this is the value of  $A_0$  in the formula.

•In our first calculated example, we determined the radioactive decay constant for  $^{32}$ P was 4.8472  $\cdot$  10<sup>-2</sup> per day; this is the value of  $\lambda$ .

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•We have decided to estimate the radioactivity remaining after 143 days; this is the value of *t*.

•The equation will solve for *A*, which is the estimated radioactivity from the spill remaining throughout campus after the time interval of 143 days.

**Step 1:** Begin with the equation:

$$A = A_0 \cdot e^{-\lambda}$$

**Step 2:** Insert the known value for initial radioactivity ( $A_0 = 3.7 \cdot 10^5$  Bq), radioactive decay constant ( $\lambda = 4.8472 \cdot 10^{-2}$  per day) and elapsed time (t = 143 days), and solve for *A*:

 $A = 3.7 \cdot 10^5 \cdot e^{-4.8472 \cdot 10^{-2} \cdot 143} = 361.3194 \,\mathrm{Bq}$ 

*Note:* If you do not get the correct answer, be sure you entered the exponents with the correct signs. Most calculators have a function key to change the sign of numbers from positive to negative, often designated with a plus/ minus designation, +/-, or as a minus within parentheses, (-). Also look for a key that raises e to a power, usually designated as ex. Finally, you will use a key that raises 10 to a power, typically represented as  $10^x$ . With these buttons, the keystrokes will be similar to the following in this case (function keys are shown in brackets): 3.7  $[10^x]$  (5) X  $[e^{x}]([+/-] 4.8472 [10^{x}]([+/-]2) X 143) =.$ Alternatively, in an Excel spreadsheet, the proper formula for this example is:  $=3.7*10^{5}*EXP(-4.8472*10^{-2}*143).$ 

Step 3: The total radioactivity remaining from the <sup>32</sup>P spill after 143 days is estimated to be about 361.3194 Bq, which is about 0.1% of the initial radioactivity. Again, this estimate refers to the radioactivity remaining throughout the entire campus. The radioactivity of each footprint will be much less. If monitoring confirms the radioactivity of each footprint is indistinguishable from background levels after 143 days (10 half-lives), we will consult applicable regulations to determine whether the contamination has sufficiently decayed in place and whether any further action is required. (Note: Confirmatory measurements of remaining radioactivity would be taken after duct tape and PVC fabrics are removed from the footprints.)

Alternate example. We will now consider a different example to predict the remaining quantity of a radioisotope (*N*). The quantity of an isotope is usually specified in units of mass or as the actual

number of atoms present. Consider, for example, the unfissioned amount of plutonium-239 (<sup>239</sup>Pu) remaining from the Trinity nuclear test in 1945 (Figure 4, p. 33).

<sup>239</sup>Pu does not decay directly into a stable isotope. Instead, <sup>239</sup>Pu decays mainly by alpha emission to <sup>235</sup>U, which is also radioactive. <sup>235</sup>U then decays through a complex chain of radioactive progeny, described by calculations that are beyond the scope of this article. For this example, we will estimate the mass of <sup>239</sup>Pu from the Trinity tests that is expected to remain to date, while ignoring the mass of any radioactive progeny.

The total amount of <sup>239</sup>Pu incorporated in the Trinity Gadget has been estimated at about 6 kilograms (6 kg; Beck et al., 2020). It is believed that only about 0.8 kg of <sup>239</sup>Pu was fissioned during the explosion, leaving about 5.2 kg of unfissioned <sup>239</sup>Pu that vaporized and was deposited on the surface of the earth as fallout. Other radioactive elements resulted from the explosion and its aftermath; however, we will consider only the fate of the 5.2 kg of unfissioned <sup>239</sup>Pu.

Since we are estimating the amount of remaining  $^{239}$ Pu in kg, we will solve for *N* based on the following data:

•The unfissioned <sup>239</sup>Pu was about 5.2 kg; this is the value of  $N_0$  in the formula.

•In the second calculated example, we determined the radioactive decay constant for <sup>239</sup>Pu is about 2.8761  $\cdot$  10<sup>-5</sup> per year; this is the value of  $\lambda$ .

•We will estimate the amount of Trinity <sup>239</sup>Pu remaining after 77.3 years, which is the interval from the date of the test (July 16, 1945) until the present time of Nov. 1, 2022; this is the value of *t*.

•The equation will solve for *N* in kg, which is the estimated amount of Trinity <sup>239</sup>Pu remaining after 77.3 years.

**Step 1:** Begin with the equation:

 $N = N_0 \cdot e^{-\lambda \cdot t}$ 

**Step 2:** Insert the known value for initial quantity of unfissioned <sup>239</sup>Pu ( $N_0$  = 5.2 kg), radioactive decay constant ( $\lambda$  = 2.8761 · 10<sup>-5</sup> per year) and elapsed time (t = 77.3 years), and solve for *N*:

 $N = 5.2 \cdot e^{-2.8761 \cdot 10^{-5} \cdot 77.3} = 5.1885 \text{ kg}$ 

**Step 3:** The calculation indicates the total mass of unfissioned <sup>239</sup>Pu remaining from the blast (5.1885 kg) has barely changed during the 77.3 years that have elapsed. The small change is due to the long half-life (and the resulting small decay constant) of <sup>239</sup>Pu. The <sup>239</sup>Pu that has decayed (0.0115 kg) was transformed

into radioactive <sup>235</sup>U, which has its own half-life of about 700 million years and has been slowly following its own complicated radioactive decay chain since the explosion. In other words, radioactivity from the blast will remain in the environment for eons. Most of the fallout from the Trinity test landed near the site of the explosion. The rest was lofted higher in the atmosphere and landed farther away. In fact, a small percentage was distributed globally throughout the surface of the earth.

#### You Do the Math

Apply your knowledge to the following questions. Answers are on p. 41.

3. Remember that radioactive <sup>210</sup>Po is used as a static eliminator in textile mills and other industries. We'll imagine a static elimination device (with <sup>210</sup>Po incorporated in a foil strip) is used on an industrial printing press. Further imagine that the radioactivity of the <sup>210</sup>Po device was about 6.5 billion (6,500,000,000) Becquerel  $(6.5 \cdot 10^9 \text{ Bq})$  at the time of manufacture. Using the value of the <sup>210</sup>Po radioactive decay constant,  $\lambda$ , that you calculated in "You Do the Math" question 1, determine the radioactivity (in Bq) expected after 365 days from the manufacture date. Since you calculated the decay constant in days, be sure to use days for the elapsed time in the equation:

$$A = A_0 \cdot e^{-\lambda}$$

4. Remember that radioactive <sup>14</sup>C decays into nonradioactive <sup>14</sup>N. The ratio of naturally occurring 14C to 14N can be used to determine the age of organic materials that are up to 50,000 years old. Imagine an animal died 25,000 years ago. Also imagine a tiny bone from the animal contained 1 billion (1,000,000,000 or  $1 \cdot 10^9$ ) atoms of <sup>14</sup>C at the time of death. Using the <sup>14</sup>C radioactive decay constant,  $\lambda$ , that you calculated for "You Do the Math" question 2, determine the number of <sup>14</sup>C atoms expected to remain in the preserved bone after 25,000 years. Since you calculated the decay constant in years, be sure to use years for the elapsed time in the equation:

$$\mathbf{V} = N_0 \cdot e^{-\lambda \cdot t}$$

#### **Concluding Comments**

Our equations predict the quantity (in mass or number of atoms) or radioactivity (in Bq or Ci) after a period of radioactive decay. Keep in mind the original radioactive atoms do not cease to exist; instead, they are transformed into other isotopes or elements that may or may not be radioactive themselves. For substances with complex decay chains, more advanced equations are required to estimate the total radioactivity (i.e., activity of the original isotope and progeny) at any time. Also remember the equations produce estimates, so measurements must be taken after a period of decay to confirm the residual radioactivity.

Finally, readers may have already surmised that we can combine equations for the radioactive decay constant and residual radioactivity/quantity into single equations, as follows:

$$A = A_0 \cdot e^{-\frac{\ln(2)}{t_{1/2}} \cdot t}$$
$$N = N_0 \cdot e^{-\frac{\ln(2)}{t_{1/2}} \cdot t}$$

These equations will estimate the remaining radioactivity or quantity directly from the half-life, without calculating the decay constant as an initial step. This is a bit more cumbersome when using a handheld calculator, but the result is more accurate because rounding errors will be minimized. As an example, consider the marginally increased accuracy when we use this method to recalculate residual radioactivity from the <sup>32</sup>P spill in Figure 1 (p. 30), where  $A_0 = 3.7 \cdot 10^5$ Bq;  $t_{\frac{1}{2}} = 14.3$  days; and t = 143 days:  $A = 3.7 \cdot 10^5 \cdot e^{-\frac{ln(2)}{14.3} \cdot 143} = 361.3281$  Bq

#### How Much Have I Learned?

Try these problems on your own. Answers are on p. 41.

5. Radioactive cobalt-60 (60Co) is often used in industrial irradiation equipment to sterilize medical products, foods, and other items prior to sale. 60Co emits beta and gamma radiation to become stable nickel-60 (60Ni). The half-life of 60Co is about 5.3 years. Imagine an irradiator facility with a radioactive source that contained 9.8 grams (g) of pure <sup>60</sup>Co on Nov. 1, 2019. Answer the following:

a. Since the half-life is expressed in years, calculate the <sup>60</sup>Co radioactive decay constant,  $\lambda$ , per year, using the equation:

$$\lambda = \frac{\ln(2)}{t_{1/2}}$$

b. Using the value of the decay constant,  $\lambda$ , that you calculated for question 5a, determine the mass (in g) of  $\overline{^{60}}$ Co that would remain from the original 9.8 g after 3 years, from Nov. 1, 2019, to Nov.

1, 2022. Since you calculated the decay constant in years, be sure to use years for the elapsed time in the equation:

$$N = N_0 \cdot e^{-\lambda \cdot t}$$

6. Radioactive sodium-24 (<sup>24</sup>Na) is sometimes used for leak detection in industrial pipelines. <sup>24</sup>Na emits beta and gamma radiation to become stable magnesium-24 (<sup>24</sup>Mg). The half-life of <sup>24</sup>Na is about 15 hours. Imagine 3.7 · 1010 Bq of <sup>24</sup>Na is injected into a large underground pipeline during a search for leaks. Answer the following:

a. Since the half-life is expressed in hours, calculate the <sup>24</sup>Na radioactive decay constant,  $\lambda$ , per hour using the equation:

$$\lambda = \frac{\ln(2)}{t_{1/2}}$$

b. Using the value of the radioactive decay constant,  $\lambda$ , that you calculated in question 6a, determine the radioactivity (in Bq) that would remain from the original  $3.7 \cdot 10^{10}$  Bq of <sup>24</sup>Na at an elapsed time of 48 hours after the injection. Since you calculated the decay constant in hours, be sure to use hours for the elapsed time in the equation:

 $A = A_0 \cdot e^{-\lambda \cdot t}$ 

#### The Language of Radioactive Decay

You may encounter the following concepts in certification exams and conversations with engineers and healthcare professionals. Match the numbered concepts with their paraphrased definitions (lettered). All concepts have been defined in the text, formulas and illustrations. Answers are on p. 41.

#### Concepts

- 7. alpha particle 8. beta particle 9. electron 10. element 11. gamma ray 12. half-life 13. ion 14. ionizing radiation 15. isotope 16. neutron
- 17. nucleus
- 18. proton
- 19. radioactive decay

#### Definitions (in random order)

a. amount of time it takes, on average, for half the atoms of an isotope to decay into a different isotope or element

b. electromagnetic radiation (pure energy) emitted from the nucleus of an unstable atom

c. general term for changes that tend to stabilize the nucleus of an unstable (radioactive) atom

d. interior of an atom, where protons and neutrons are located

e. molecules, atoms and subatomic particles with a nonneutral (positive or negative) charge

f. negatively charged particle (negatron) or positively charged particle (positron) that is emitted from the nucleus of an unstable atom

g. negatively charged particles that exist in a cloud surrounding the nucleus of an atom

h. particle consisting of two protons and two neutrons that is emitted from the nucleus of an unstable atom

i. particle with a neutral electrical charge located within the nucleus of an atom

j. positively charged particle located within the nucleus of an atom

k. radiation that can create ions in other materials

l. type of atom (such as uranium, U) that is defined by the number of protons in its nucleus

m. type of atom (such as uranium-235, <sup>235</sup>U) that is defined by the number of protons plus the number of neutrons in its nucleus **PSJ** 

#### References

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#### Irmo, SC

12/20: Introduction to Incident Investigation. Southeastern OSHA Training Institute Education Center; (800) 227-0264; www.ies.ncsu.edu/otieducationcenter.

#### Houston, TX

12/23: Competent Person Inspection. Scaffold Training Institute; (281) 332-1613; www.scaffoldtraining.com.

#### Vienna, Austria & Online

12/29-12/30: International Conference on Public Health and Occupational Medicine. World Academy of Science, Engineering and Technology. https:// waset.org.

### **JANUARY 2023**

#### Des Moines, IA & Online

1/5: Job Safety Analysis. Illinois-Iowa Safety Council. (515) 276-4724; www .iisc.org.

#### Online

1/9: Lockout/Tagout. Texas A&M Engineering Extension Service; (800) 723-3811; www.teex.org/prt.

#### Kennewick, WA

1/9-1/11: Supervisory Safety and Health Duties. Northwest Center for Occupational Health and Safety; (800) 326-7568; http://nwcenter.washington.edu.

#### Online

1/9-1/12: Electrical Standards. OSHA Training Center Chabot-Las Positas Community College District; (866) 936-6742; https://osha4you.com.

#### Portland, OR & Online

1/9-1/13: Occupational Hearing Conservationist. Audiology Consulting and Training; (425) 757-7801; https://audiolo gycat.com.

#### Online

1/9-1/13: Radiation Safety Officer. NV5; (954) 495-2112; www.nv5.com.

#### Online

1/10-1/12: Fall Protection. University of Texas at Arlington OSHA Education Center; (866) 906-9190; https://web-ded .uta.edu.

#### Online

1/10-3/2: Certified Industrial Hygienist. Bowen EHS Inc.; (866) 264-5852; www .bowenehs.com.

#### Online

1/11: Developing a Driver Safety Program. Ohio Bureau of Workers' Compensation, Division of Safety and Hygiene; (800) 644-6838; https://info .bwc.ohio.gov.

#### Online

1/12 • Outcome-Focused Learning: Strategies for the Design, Delivery and Evaluation for Virtual Environments. Understand adult learning principles, how to make virtual training more engaging and training criteria outlined in the ANSI/ASSP Z490.2 standard. ASSP Blacks in Safety Excellent Common Interest Group; https://bit.ly/3s2uapS.

#### Online

1/12: Cal/OSHA Compliance. Pryor Learning; (800) 780-8476; www.pryor.com.

#### Sandy, UT

1/12: Occupational Health Hazards for the Safety Manager. WCF Insurance; (800) 446-2667; www.wcf.com.

#### St. Pete Beach, FL

1/15-1/18: 2023 FDSOA Health and Safety Conference. Fire Department Safety Officers Association; (248) 880-1864; https://fdsoa.org.

### San Diego, CA

1/17-1/19: MCAA 2023 Safety Conference. Mechanical Contractors Association of America; (301) 869-5800; www .mcaa.org.

#### New Orleans, LA

1/18-1/20: AGC Construction Safety and Health Conference. The Associated General Contractors of America; (703) 548-3118; https://safety.agc.org.

#### New Orleans, LA

1/18-1/20: ACI 2023 Risk Management Conference. Airports Council International—North America; (202) 293-8500; https://airportscouncil.org.

#### Madison, WI

1/18-1/20: Process Safety Management Audits for Compliance and Continuous Safety Improvement. University of Wisconsin; (800) 462-0876; http://epdweb .engr.wisc.edu.

#### Osage Beach, MO

1/19-1/20: 2023 Missouri Mine Safety and Health Conference. Missouri Mine Safety and Health Conference; (573) 635-0208; https://mmshc.org.

#### Vacaville, CA

1/23-1/24: Spirometry. NIOSH; (800) 232-4636; www.cdc.gov/niosh.

#### Larned, KS

1/23-2/27: Hazardous Materials Operations. University of Kansas Continuing Education; (877) 404-5823; http://kupce .ku.edu.

#### Bend, OR

1/30-1/31: Mid-Oregon Construction Safety Summit. Central Oregon Safety and Health Association; (541) 332-7104; www.cosha.org.

#### Math Toolbox, continued from pp. 30-35

## Answers: The Case of the Radioactive Footprints

#### You Do the Math

Your answers may vary slightly due to rounding.

- 1.  $\lambda = \frac{ln(2)}{138.38} = 5.0090 \cdot 10^{-3}$  per day
- 2.  $\lambda = \frac{ln(2)}{5,700} = 1.2160 \cdot 10^{-4}$  per day

- 3.  $A = 6.5 \cdot 10^9 \cdot e^{-5.0090 \cdot 10^{-3} \cdot 365}$ = 1.0445 \cdot 10^9 Bq
- 4.  $N = 1 \cdot 10^9 \cdot e^{-1.2160 \cdot 10^{-4} \cdot 25,000}$ = 4.7835 \cdot 10^7 C<sup>14</sup> atoms

### How Much Have I Learned?

Your answers may vary slightly due to rounding.

5a. 
$$\lambda = \frac{ln(2)}{5.3} = 1.3078 \cdot 10^{-1}$$
 per year

5b.  $N = 9.8 \cdot e^{-1.3078 \cdot 10^{-1} \cdot 3} = 6.6196 \text{ g}$ 

6a. 
$$\lambda = \frac{ln(2)}{15} = 4.6210 \cdot 10^{-2}$$
 per hour

6b.  $A = 3.7 \cdot 10^{10} \cdot e^{-4.6210 \cdot 10^{-2} \cdot 48}$ = 4.0263 \cdot 10<sup>9</sup> Bq

The Language of Radioactive Decay 7. h; 8. f; 9. g; 10. l; 11. b; 12. a; 13. e; 14. k; 15. m; 16. i; 17. d; 18. j; 19. c.